XIII. *On the Constitution of the Solid Crust of the Earth.*

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Introduction.

A FEW years ago I proposed the following hypothesis regarding the Constitution of the Earth's Solid Crust, viz.:—that the variety we see in the elevation and depression of the earth's surface, in mountains and plains and ocean-beds, has arisen from the mass having contracted unequally in becoming solid from a fluid or semifluid condition*: and that

* I first proposed this hypothesis in a paper printed in the Proceedings of the Royal Society, No. 64, 1864; see pp. 270–276 of that paper; and afterwards in the third edition of my 'Figure of the Earth,' pp. 134–137.

Mr. AIRY was the first to suggest, in Phil. Trans. 1855, p. 101, a deficiency of matter below mountain-regions; and he there pointed out that such a deficiency would counteract in great measure the effect of the Himalayas themselves on the plumb-line, the attraction of which, I had shown in a previous paper in the same volume, by direct calculation, would be considerable and would introduce new anomalies. The reasoning, however, by which he proceeded to show that this deficiency must exist involved conditions which appeared to me inadmissible—viz. (1) that the solid crust is comparatively thin, and (2) that the density of the solid crust is less than that of the lava on which it was supposed to float. See my remarks on Mr. AIRY's paper at pp. 51, 52, Phil. Trans. 1856; in which also I give reasons for not admitting, what his data require, that the present form of the surface has arisen solely or mainly from hydrostatic principles. This hypothesis of deficiency of matter, as there advanced, does not appear to rest on any true physical basis.

In the Phil. Trans. 1858, p. 745, following up Mr. AIRY's suggestion of deficiency of matter—but not as he conceived it to exist, in a thin crust, immediately below the mountain-mass, and by buoyancy supporting the crust by the principle of floatation—I proposed the hypothesis of the mountain-mass having been formed by upheaval, by a slight expansion of the solid crust, and a corresponding attenuation of its density, from a great depth below (par. 4, p. 747); and I showed by calculation, in that paper, that the resulting effect of such attenuation on the plumb-line would be considerable, and quite comparable with the effect which calculation showed would be produced by the mountains themselves.

Immediately after this another source of disturbance of the plumb-line suggested itself to me, viz. deficiency

below the sea-level under mountains and plains there is a deficiency of matter, approximately equal in amount to the mass above the sea-level; and that below ocean-beds there is an excess of matter, approximately equal to the deficiency in the ocean when compared with rock; so that the amount of matter in any vertical column drawn from the surface to a level surface below the crust is now, and ever has been, approximately the same in every part of the earth.

2. The process by which I arrived at this hypothesis I will explain. In the Philosophical Transactions for 1855 and 1858 I showed that the Himalayas and the Ocean must have a considerable influence in producing deflection of the plumb-line in India. But by a calculation of the mean figure of the earth, taking into account the effect of local attraction, it appeared that nowhere on the Indian Arc of meridian through Cape Comorin is the resultant local attraction, arising from all causes, of great importance*. This result at once indicated that in the crust below there must be such variations of density as nearly to compensate for the large effects which would have resulted from the attraction of the mountains on the north of India and the vast ocean on the south, if they were the sole causes of disturbance,—and that, as this near compensation takes place all down the arc, nearly 1500 miles in length, the simplest hypothesis is, that beneath the mountains and plains there is a deficiency of matter nearly equal to the mass above the sea-level, and beneath ocean-beds an excess of matter nearly equal to the deficiency in the ocean itself.

3. The compensation, should the hypothesis be true, is not complete, but approximate;

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of matter in the ocean; and in a paper in the same volume of the Phil. Trans. (p. 779) I showed by calculation that the vast ocean stretching down to the south pole would produce considerable effects in the southern parts of India, such as the Survey altogether failed to detect. This seemed to imply that, as beneath mountain-regions there is a deficiency of matter, so beneath ocean-beds there must be an excess, in order to account for the deficiency in the effect of the ocean (which of itself would be large) not being discernible. The thought of an excess of matter below the ocean-bed accords with the remark which Sir John Herschel once made, that the ocean-bed of the Pacific must be more dense than the average surface of the solid crust, otherwise the protuberant ocean would be drawn away and would flow to other parts of the surface.

My calculations had shown, then, that a considerable effect on the plumb-line must result from each of the following causes taken separately:—(1) the mountain-region, (2) the ocean, (3) any widespread, though slight, deficiency or excess of matter in the solid crust. As there is reason to believe that resultant local attraction is nowhere in India very great, and is generally small, it must follow that, generally speaking, below high ground there is a deficiency of matter, and below ocean-beds an excess.

But there had not been at that time any physical hypothesis proposed to account for these two conditions of the crust, and to connect them together as results of one and the same cause. An hypothesis was, however, suggested in 1864, in my paper in the 'Proceedings of the Royal Society' alluded to at the beginning of this note, and referred to in the text, viz. that all the varieties we see in the earth's surface (in mountains, plains, and ocean-beds) have arisen from the earth's mass having contracted unequally in a vertical direction, in passing from a fluid to a solid state—a necessary result of its fluid origin being that the amount of matter in any vertical column down to a level surface is the same and always has been the same, whatever the changes in length it may have undergone.—Calcutta, April 6, 1871.

* See Proc. Royal Soc. No. 64, 1864; but especially Phil. Mag. January and February, 1867.

for deflections of the plumb-line do exist—such, for instance, as that near Moscow. Thus also in India a comparison of the amplitudes of arcs obtained by observation and by calculation shows the same. I have constructed the following Table to bring this to view. The data in the 1st, 2nd, and 3rd columns of numbers are derived from the Chapter on the Figure of the Earth in the volume of the British Ordnance Survey.

TABLE I.

Stations.	Observed latitudes.	Measured distances between successive stations, in feet.	Amplitudes between successive stations.		Difference, or relative deflection of plumb-line.	Deflections of plumb-line relative to Punnae.	Equivalent horizontal meridian force. + means south.
			Observed or astronomical.	Calculated: $a=20926184$, $b=20855301$.			
	° ′ ″		° ′ ″	° ′ ″	″	″	
Punnae	8 9 31·132	....	....	....	....	....	....
Retchapolliam	10 59 42·276	1029173·7	2 50 11·14	2 50 10·32	+0·82	+0·82	+0·0000040 g
Dodagoontah	12 59 52·165	727386·3	2 0 9·89	2 0 15·19	−5·30	−4·48	−0·0000217 „
Namthabad	15 5 53·562	761813·4	2 6 1·40	2 5 55·48	+5·92	+1·44	+0·0000070 „
Damargida	18 3 15·292	1073410·9	2 57 21·73	2 57 23·41	−1·68	−0·24	−0·0000012 „
Takal Khera	21 5 51·532	1105539·8	3 2 36·24	3 2 38·56	−2·32	−2·56	−0·0000124 „
Kalianpur	24 7 11·262	1097364·9	3 1 19·73	3 1 13·62	+6·11	+3·55	+0·0000172 „
Kaliana	29 30 48·322	1961138·0	5 23 37·06	5 23 41·65	−4·59	−1·04	−0·0000050 „

I have calculated the amplitudes by means of the formula

$$\lambda=\frac{2s}{a+b}\left\{1+\tfrac{3}{2}\varepsilon\cos 2m\right\}.$$

For finding such very small angles as the deflections, which are the differences of very much larger angles, no doubt the introduction of the square of the ellipticity would slightly modify the results in the last three columns; but not so as to affect the use I shall make of them. It is seen from this Table (last column but one) that the deflections of the plumb-line, though small, are yet sensible quantities; and they do not correspond with the heights of the neighbourhood of the several stations.

The hypothesis, therefore, is not exact, but only approximately true, when applied generally, on a large scale. This, indeed, we might anticipate for other reasons. For example, if the crust below the ocean-beds has contracted or expanded at all (which no doubt it has) since it became too thick* to be able to adjust itself as it floated upon the

* The late Mr. HOPKINS pointed out that the amount of precession in the earth's axis, caused by the disturbing force of the sun and moon, would be very different in amount as the solid crust was thin or thick; and he made a calculation (Philosophical Transactions, 1839, 1840, 1842) based upon this idea, and showed that the crust must now be at least 1000 miles thick. M. DELAUNAY has lately read a paper before the Academy of Sciences controverting Mr. HOPKINS's idea—saying that the interior fluid must long ago have conformed to the motion of the crust in consequence of friction and viscidity, and be now moving with it as if the whole were solid. If the crust moved round a steady axis, this might be true. But this is not the case. The force which causes precession is continually tending to draw the earth's pole towards the pole of the ecliptic—but does not move it in that direction, but, combining with the rotatory motion, causes it to shift through a small angle at right angles to the line joining the two poles. The extent of the angle must depend upon the force, the length of the infinitesimal portion of time, and the moment of inertia of the crust; for the fluid, during this infinitesimal portion of time, will not have been able to acquire the new motion; the crust, having no solid connexion with the fluid, will slip over it, with a twist. Suppose even that at the present instant the fluid were

fluid below, and if this contraction or expansion was different from that of the dry land, water would flow in or out of the ocean and disturb the exact equality of matter in any two vertical columns drawn down from the surface of the land and the water. Also, as the crust contracted and brought into play the prodigious force of compression, which would inevitably cause the crust to give way at the weakest part and produce anticlinal lines, crushing, sliding, and interpenetration, there would be a slight increase of mass in some parts on this account. From these and similar causes it is readily seen that, since the epoch when the crust ceased to be thin enough to adjust its own position according to varying circumstances, changes must have occurred which would modify the previous state of things. Still the result of these modifying causes must be but slight, compared with those large effects of the mountains and ocean and crust, which nearly compensate each other, and which suggest the hypothesis. Another cause arises from the calculation itself. It is necessary to assume some law of distribution of the mass, that the calculation may be possible. I assume that the deficiency or excess of matter is distributed *uniformly* to a depth bearing a fixed ratio to the height of the land or the depth of the ocean. The actual distribution most likely differs from this. But this is taken as an average. We must expect, for these reasons, to find that the hypothesis is not satisfied with exact precision.

4. Colonel J. T. Walker, R.E., Superintendent of the Great Trigonometrical Survey of India, to whom I had communicated the formulæ developed in this paper, has lately supplied me with information showing the results of the Pendulum Observations recently made along the Great Indian Arc of meridian and at other places, and has obligingly allowed me to make use of the data. While these observations have been going on I have looked forward with great interest to the results, as I felt persuaded that the observations would furnish me with the means of testing, in a new and independent way, the truth of my hypothesis regarding the constitution of the earth's crust. It is the object of the present communication to show with what measure of success the test has been applied*.

moving exactly as the crust, the force producing precession would from this instant give the crust a new motion, which the fluid has not, and which it has not time to acquire before, in the next small portion of time, the crust has shifted again with the same twist. The amount of precession must therefore depend upon the moment of inertia during the time it is generated, and therefore upon the thickness of the crust and not at all upon the fluid.

* Since the above was written, Colonel Walker has sent me a copy of a printed letter and Note on the Pendulum Observations, from which I extract the following remarks. "The observations at the five northernmost " stations indicate that there is much probability that the density of the strata of the earth's crust under and " in the vicinity of the Himalayan mountains is less than that under the plains to the south, the deficiency " increasing as the stations approach the Himalayas, and being greatest when they are north of the Siwaliks. " On the other hand, the observations of the five southernmost stations show an increase of density in proceeding " from the interior of the peninsula to the coast at Cape Comorin. Thus both groups of observations tend to " confirm the hypothesis that there is a diminution of density in the strata of the earth's crust under mountains and continents, and an increase of density under the bed of the ocean." This is the hypothesis I published in 1864: see Proceedings, No. 64.

§ 1. *Data regarding Pendulum Observations in India.*

5. From the information furnished me by Colonel Walker I complete Table II. I have selected five stations on the Arc of Meridian at nearly the same distances from one another in succession, averaging about 5° 15′ apart. I have also taken stations on the coast and on an island (Minicoy). Punnae is close to Cape Comorin.

Table II.					Table III.			
Stations.	Geodetic coordinates. North latitude.	Geodetic coordinates. East longitude.	Geodetic coordinates. Heights, in feet.	Observed numbers of vibrations at stations, reduced for thermometer and barometer.	Relative gravity.	Reduction for height above sea.	Allowance for latitude.	Relative gravity freed from effects of height and latitude.
Indian Arc Stations.								
Punnae	8° 10′	77° 41′	44	85978·18	1−0·0005074	+ 42	− 1062	1−0·0006094
Bangalore	13 4	77 37	3007	74·63	1−0·0005900	+2879	− 2689	1−0·0005710
Damargida	18 3	77 43	1934	86·16	1−0·0003219	+1852	− 5050	1−0·0006417
Kalianpur	24 7	77 42	1765	86005·76	1+0·0001340	+1690	− 8783	1−0·0007753
Kaliana	29 31	77 42	826	22·26	1+0·0005177	+ 791	−12769	1−0·0006801
Coast Stations.								
Punnae	8 10	77 41	44	85978·18	1−0·0005074	+ 42	− 1062	1−0·0006094
Alleppy	9 30	76 20	6	81·23	1−0·0004365	+ 6	− 1433	1−0·0005792
Mangalore	12 52	74 49	7	84·27	1−0·0003658	+ 7	− 2609	1−0·0006260
Madras	13 4	80 17	27	84·40	1−0·0003628	+ 26	− 2689	1−0·0006291
Cocanada	16 56	82 18	9	93·56	1−0·0001498	+ 9	− 4463	1−0·0005952
Ocean Station.								
Minicoy	8 17	73 2	6	82·31	1−0·0004114	+ 6	− 1092	1−0·0005200

6. The numbers of vibrations of the pendulum at the several stations differ from each each other for three reasons. The stations differ (1) in latitude, (2) in height above the sea, (3) in local attraction. Were all these allowed for, the numbers would come out the same for the several stations. In Table III. I allow for the influence of the first and second of these causes; and the last column shows by its variations what we have to attribute to the third cause, viz. local attraction.

The numbers in the first column of Table III. are obtained from those of the last column of Table II. by dividing them all by 86000, and then doubling the small (that is, the decimal) part, because gravity varies as the square of the number of vibrations. In the second and third columns of Table III. the numbers are the last of seven places of decimals, the ciphers and the decimal point being omitted for convenience. The numbers in the third column are obtained by means of Clairaut's Theorem, which shows that gravity varies as $1+(\frac{5}{2}m-\varepsilon)\sin^2 l$; the ellipticity being $\frac{1}{295}$.

§ 2. *Data regarding the contour of the Continent of India.*

7. Viewing the continent of India generally, and allowing for ridges of hills and hollows, it may be said to lie more or less evenly, as far as this problem is concerned, up to the foot of the Himalayas. With regard to the Himalayas, by a careful examination of published documents and maps of the Great Trigonometrical Survey, and of Colonel R. Strachey's map referred to in the Philosophical Transactions for 1858, p. 774, and

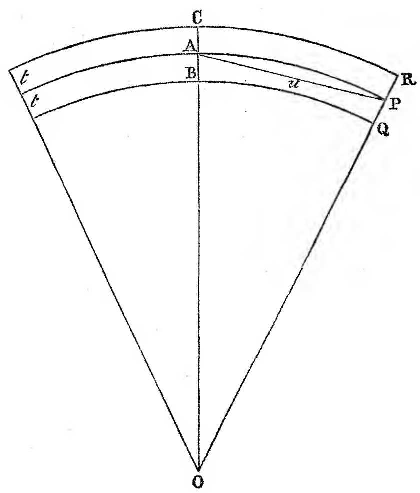
by laying down the heights on a plan, I come to the conclusion that the mass of the Himalayas may, for the purpose of this problem, be represented as a vast tableland, 15,000 feet or 2·48091 miles above the sea-level. If a number of zones are drawn around the three stations Kaliana, Kalianpur, and Damargida, their width being about 50 miles (49·45 exactly, as will be seen further on), then the tableland will begin on the 2nd zone from Kaliana, the 8th from Kalianpur, and the 17th from Damargida; and the horizontal extent of the tableland lying on these and the following zones, as determined from the plan, is shown by the values of β in the following Table IV. In some cases β is made up of two or even of three portions added together, when the zones cross the tableland in two or more places, owing to its irregular outline.

TABLE IV.

Kaliana.		Kalianpur.		Damargida.	
Zones.	β.	Zones.	β.	Zones.	β.
2	74°	8	45°	17	22°
3	110	9	62	18	41
4	127	10	79	19	50
5	137	11	88	20	62
6	144	12	92	21	68
7	149	13	97	22	73
8	113	14	76	23	58
9	88	15	58	24	46
10	79	16	47	25	32
11	66	17	46	26	9
12	56	18	35	27	12
13	109	19	76	28	12
14	87	20	46	29	12
15	79	21	6		
16	61	22	6		
17	37				

§ 3. *Formulæ for the Vertical Attraction of a Spherical Cap of matter on the earth's surface on the mid points of its upper and lower surfaces, and of its divisions into a Central Portion and Zones.*

8. By a spherical cap is meant such a part of a spherical shell as would be generated by the revolution of the figure A P Q B round the vertical A B.



A is the station attracted. The chord A P$=u$ miles; the thickness of each cap, above and below the station-level, is t miles. Let c and r be the distances of the attracted point A and of any particle of the cap from O; θ the angle between c and r; $z=c-r$; $u=c$ chord θ; $v=c$ vers θ, which $=u^2 \div 2c$.

FIRST. Suppose the cap immediately *below* the station-level. The attraction of an elementary ring

of matter round A, reckoned positive downwards,

$$=2\pi r \sin\theta \,.\, dz \,.\, rd\theta \,.\, \varrho \frac{c-r\cos\theta}{(c^2+r^2-2cr\cos\theta)^{\frac{3}{2}}}=\frac{2\pi\varrho r^2}{c^2}\frac{d}{d\theta}\left(\frac{r-c\cos\theta}{\sqrt{c^2+r^2-2cr\cos\theta}}\right)d\theta dz$$

$$=\frac{2\pi\varrho}{c^2}(c-z)^2\frac{d}{d\theta}\left(\frac{2c\sin^2\frac{1}{2}\theta-z}{\sqrt{z^2+4c(c-z)\sin^2\frac{1}{2}\theta}}\right)d\theta dz.$$

Integrating from $\theta=0$ to $\theta=\theta$, and then putting $2c\sin\frac{1}{2}\theta=u$ and $u^2=2cv$, Total Attraction of the Cap

$$=\frac{2\pi\varrho}{c^2}\int_0^t\left\{(c-z)^2+\frac{(v-z)(c-z)^2}{\sqrt{z^2+2(c-z)v}}\right\}dz$$

$$=\frac{2\pi\varrho}{c^2}\left\{\frac{c^3-(c-t)^3}{3}+\int_0^t\frac{(v-z)(c-z)^2dz}{\sqrt{z^2-2vz+2cv}}\right\}.$$

Integrating by parts, the integral becomes

$$=-(c-z)^2\sqrt{z^2-2vz+2cv}-2\int(c-z)\sqrt{z^2-2vz+2cv}\,.\,dz$$

$$=-(c-z)^2\sqrt{z^2-2vz+2cv}+\tfrac{2}{3}(z^2-2vz+2cv)^{\frac{3}{2}}+2(v-c)\int\sqrt{z^2-2vz+2cv}\,.\,dz$$

$$=-(c-z)^2\sqrt{z^2-2vz+2cv}+\tfrac{2}{3}(z^2-2vz+2cv)^{\frac{3}{2}}$$
$$-(v-c)\{(v-z)\sqrt{z^2-2vz+2cv}-(v^2-2cv)\log_e(v-z+\sqrt{z^2-2vz+2cv})\}$$

$$=-\{\tfrac{1}{3}z^2-(c-\tfrac{1}{3}v)z-c^2-\tfrac{7}{3}cv+v^2\}\sqrt{z^2-2vz+2cv}$$
$$+(v-c)(v^2-2cv)\log_e(v-z+\sqrt{z^2-2vz+2cv}).$$

Putting this for the integral, replacing $2cv$ by u^2, and taking the limits, Vertical Attraction of the Cap below the station-level

$$=\frac{2\pi\varrho c}{3}\left\{1-\left(1-\frac{t}{c}\right)^3+\frac{3u}{c}-\frac{7u^3}{2c^3}+\frac{3u^5}{4c^5}\right.$$
$$-\left[\frac{3u}{c}-\frac{7u^3}{2c^3}+\frac{3u^5}{4c^5}-\left(\frac{3u}{c}-\frac{u^3}{2c^3}\right)\frac{t}{c}+\frac{u}{c}\frac{t^2}{c^2}\right]\sqrt{\frac{t^2+u^2}{u^2}-\frac{t}{c}}$$
$$\left.+3\left(\frac{u^2}{2c^2}-1\right)\left(\frac{u^4}{4c^4}-\frac{u^2}{c^2}\right)\log_e\frac{\frac{u}{2c}-\frac{t}{u}+\sqrt{\frac{t^2+u^2}{u^2}-\frac{t}{c}}}{\frac{u}{2c}+1}\right\}.$$

This is the exact expression.

9. SECONDLY. Suppose the Cap is immediately *above* the station-level. The above formula requires in this case some modification. In the first place, when the limits of θ are taken and θ is put$=0$, the radical in the denominator is now $-z$, and not z as before. This will change the sign of the first term $(c-z)^2$ in the integral with regard to θ, and will change the signs of the first and second terms within the brackets of the final integral. Again, the limits of integration with regard to z must be taken from $z=-t$ to $z=0$, which is the same as putting $-t$ for t and also changing the sign of every term of the

final integral. Making these changes, and still estimating, as I always shall do, the attraction positive downwards,

Vertical Attraction of the Cap above the station-level

$$=\frac{2\pi\varrho c}{3}\left\{1-\left(1+\frac{t}{c}\right)^3-\frac{3u}{c}+\frac{7u^3}{2c^3}-\frac{3u^5}{4c^5}\right.$$

$$+\left[\frac{3u}{c}-\frac{7u^3}{2c^3}+\frac{3u^5}{4c^5}+\left(\frac{3u}{c}-\frac{u^3}{2c^3}\right)\frac{t}{c}+\frac{u}{c}\frac{t^2}{c^2}\right]\sqrt{\frac{t^2+u^2}{u^2}+\frac{t}{c}}$$

$$\left.-3\left(\frac{u^2}{2c^2}-1\right)\left(\frac{u^4}{4c^4}-\frac{u^2}{c^2}\right)\log_e\frac{\frac{u}{2c}+\frac{t}{u}+\sqrt{\frac{t^2+u^2}{u^2}+\frac{t}{c}}}{\frac{u}{2c}+1}\right\}.$$

As these formulæ are to be applied to find the vertical attraction of the superficial portions of the earth, it may be here stated that, as the attractions will be always small quantities, the earth may be regarded as a sphere, and c taken equal to the mean radius 3956 miles, as the height or depth of any Cap above or below the sea-level will be comparatively small.

10. These formulæ may be much reduced for use by approximation. The square of $t\div c$ will be neglected; for the greatest value $t\div c$ will have in this paper will be $1\div 13$; and therefore its square will be $1\div 169$. Expanding, then, in powers of $t\div c$ and neglecting its square, and observing that as c^2 occurs in the denominator of every term of the coefficient of the log., we may neglect t^2 everywhere in the log. itself, we have

Vertical Attraction of the Cap below the station-level

$$=2\pi\varrho\left\{t+u-\frac{7u^3}{6c^3}+\frac{u^5}{4c^5}-\sqrt{t^2+u^2}+\frac{7u^3}{6c^3}-\frac{u^5}{4c^5}+\frac{ut}{c}-\frac{u^3t}{6c^3}\right.$$

$$\left.+\frac{ut}{2c}-\frac{7u^3t}{12c^3}+\frac{u^5t}{8c^5}+\left(\frac{u^2}{2c^2}-1\right)\left(\frac{u^4}{4c^4}-\frac{u^2}{c^2}\right)\log_e\left(1-\frac{t}{u}\right)\right\}$$

$$=2\pi\varrho\left(u+t-\sqrt{u^2+t^2}+\frac{ut}{2c}\right).$$

If we take the density of the surface to be half the mean density of the earth, and g be gravity, then

Vertical Attraction of a Cap below the station-level

$$=\frac{3g}{4c}\left(u+t-\sqrt{u^2+t^2}+\frac{ut}{2c}\right). \quad \ldots\ldots\ldots \quad (1)$$

The second formula in like manner gives

Vertical Attraction of a Cap above the station-level

$$=-\frac{3g}{4c}\left(u+t-\sqrt{u^2+t^2}-\frac{ut}{2c}\right). \quad \ldots\ldots\ldots \quad (2)$$

11. The cap may be divided into Zones and a Central Portion in the following way. Let u and w be the chords of the angular distances from the station of the bounding

circles of any zone, drawn around the station on the sphere which represents the sea-level. For the zones beyond the central portion $t \div u$ is frequently so small that its fourth power may be neglected. The case where it is not so small will be considered afterwards. Then formula (1) gives for a cap of thickness t below the station-level

$$\text{Vertical Attraction} = \frac{3g}{4c}\left(t + \frac{ut}{2c} - \frac{t^2}{2u}\right);$$

and therefore for the part of the cap over the zone

$$\text{Vertical Attraction} = \frac{3g}{4c}\,\frac{w-u}{2c}\left(t + \frac{t^2 c}{uw}\right).$$

Hence, if h be the height of the station above the sea-level, the Vertical Attraction of a mass on the zone, up to the station-level

$$= \frac{3g}{4c}\,\frac{w-u}{2c}\left(h + \frac{h^2 c}{uw}\right).$$

Taking the difference of these, and putting the height of the superficial mass on the zone above the sea-level (that is, $h-t$) $=k$,

Vertical Attraction of the mass on the zone at the sea-level

$$= \frac{3g}{4c}\,\frac{w-u}{2c}\,k\left(1 + \frac{2h-k}{c}\,\frac{c^2}{uw}\right).$$

If the superficial mass rises above the station-level, we use the formula (2), which gives

$$\text{Vertical Attraction} = \frac{3g}{4c}\,\frac{w-u}{2c}\left(t - \frac{t^2 c}{uw}\right);$$

and this, added to the attraction of the mass between the sea-level and the station-level, gives, observing that in this case $k=h+t$,

Vertical Attraction of the mass on the zone at the sea-level

$$= \frac{3g}{4c}\,\frac{w-u}{2c}\left(h + t + \frac{(h^2-t^2)c}{uw}\right)$$

$$= \frac{3g}{4c}\,\frac{w-u}{2c}\,k\left(1 + \frac{2h-k}{c}\,\frac{c^2}{uw}\right),$$

precisely the same formula as before.

The formula is also true when applied to parts covered by the ocean. Let, as before, h be the height of the station above the sea-level, but k the depth of the ocean (supposed uniform under the zone). The ratio of the density of sea-water to that of rock, which equals 2·78 (half the mean density of the earth), $=0·363$. Then $h+k$ and h are heights of the station and of the surface of the attracting ocean above the level of the ocean-bed; and therefore, by means of the formula above proved,

Vertical Attraction of the ocean under the zone on the sea-level

$$= \frac{3g}{4c}\,\frac{w-u}{2c}\,0·363\,k\left(1 + \frac{2(k+h)-k}{c}.\frac{c^2}{uw}\right);$$

and therefore the effect of the *deficiency* of density of the ocean below that of rock

$$= -\frac{3g}{4c}\,\frac{w-u}{2c}\,0·637\,k\left(1 + \frac{2h+k}{2c}\,\frac{c^2}{uw}\right),$$

which is precisely the same formula as before, $-k$ being put for k because it is measured down below the sea-level, and the density being that of the deficiency of attracting matter.

Hence for all cases in which the thickness of the mass on the zone is such that the fourth power of its ratio to the mid-distance of the zone may be neglected,

Vertical Attraction of the mass on the zone

$$=\frac{3g}{4c}\frac{w-u}{2c}k\left(1+\frac{2h-k}{c}\frac{c^2}{uw}\right).$$

12. In order to simplify this as much as possible, I shall so divide the sea-level into zones that $w-u$ is the same for the central portion and each zone, equal d. Suppose n is the number of divisions—that is, a central portion, $n-2$ zones, and a central portion at the antipodes, all following the above law. Let $u_1, u_2, u_3, \ldots u_n$ be the chords to the successive bounding circles. Then

$$u_1=d,\ u_2=u_1+d=2d, \ldots\ldots\ldots u_n=nd, \text{ also } =2c;$$

$$\therefore \frac{u_1}{2c}=\frac{1}{n}, \frac{u_2}{2c}=\frac{2}{n}, \ldots \frac{u_r}{2c}=\frac{r}{n}, \ldots \frac{u_n}{2c}=\frac{n}{n}=1,$$

and the formula becomes

Vertical Attraction of the mass on the rth zone after the central portion

$$-\frac{3g}{4c}\frac{k}{n}\left(1+\frac{2h-k}{4c}\frac{n^2}{r(r+1)}\right). \qquad (3)$$

In the calculations of this paper I shall take $n=160$, which makes the radius of the central portion and the width of each zone $=49\cdot45$ miles, nearly $=50$.

I would here observe that matter may be always transferred in imagination *in azimuth* round the station without altering its effect on the vertical attraction. An application of this principle to an actual case in the earth may often assist in getting a better average for the mass. Any zone may be subdivided into smaller zones, if necessary, according to the same law. Also any zone may be divided into four-sided compartments by great circles so drawn through the station as to divide it into portions, the average heights of which may represent the mass, if it be very irregular, better than the mean height of the whole would.

If the fourth power of the thickness may not be neglected, as is done above, the formulæ (1) and (2) must be used without expanding the radical. This I shall revert to in the latter part of the next Section.

§ 4. *Formulæ for the "Resultant Vertical Attraction" of the Central Portion of the Cap, and of the Zones.*

13. By the expression "*Resultant* Vertical Attraction" of a mass I mean the vertical attraction of the mass, *diminished* by the effect of the attenuation spread uniformly below the sea-level according to the hypothesis. In the case of the ocean, the resultant vertical attraction will be the (negative) attraction of the deficiency of matter in the ocean,

increased by the attraction of the addition of matter spread uniformly through the crust below the sea-bed according to the hypothesis.

14. If the Central Portion is distributed through a depth mh below the sea-level, the Vertical Attraction of this distributed mass would equal the difference of the attraction of two caps running down to depths $(m+1)h$ and h, which, by means of the first formula in paragraph 11,

$$=\frac{3g}{4cm}\left\{\left((m+1)h+\frac{(m+1)ah}{2c}-\frac{(m+1)^2h^2}{2a}\right)-\left(h+\frac{ah}{2c}-\frac{h^2}{2a}\right)\right\};$$

and when this is subtracted from the same formula,

$$\text{Resultant Vertical Attraction of Central Portion}=\frac{3g}{4c}\frac{(m+1)h^2}{2a}.$$

Put $a=2c\div n=49{\cdot}45$, $c=3956$, and this

$$=0{\cdot}0000968h^2g \text{ or } 0{\cdot}0001917h^2g,$$

according to whether $m=50$ or 100.

If the mass on the rth zone is distributed down through a depth mh below the sea-level (that is, between the depths h and $(m+1)h$), the second formula in paragraph 11, after substituting for u and w in terms of r, gives for its effect

$$\frac{3g}{4cmn}\left\{\left((1+m)h+\frac{(1+m)^2h^2}{4r(r+1)}\frac{n^2}{c}\right)-\left(h+\frac{h^2n^2}{4r(r+1)c}\right)\right\}=\frac{3g}{4cn}\left(h+\frac{(m+2)h^2}{4r(r+1)}\frac{n^2}{c}\right).$$

Subtracting this from the same formula, h being first put for t,

$$\text{Resultant Vertical Attraction for the zone}=-\frac{3gn}{16c^2}\frac{(m+1)h^2}{r(r+1)}.$$

Put $n=160$, $c=3956$, and this

$$=0{\cdot}0000978\frac{h^2g}{r(r+1)} \text{ or } 0{\cdot}0001937\frac{h^2g}{r(r+1)},$$

according to whether $m=50$ or 100.

15. These formulæ I now tabulate for the five stations—observing that the surrounding land up to the sea stretches over radii of 0, 3, 5, 9, 14 zones respectively.

TABLE V. ($m=50$).

	Punnae.	Bangalore.	Damargida.	Kalianpur.	Kaliana.
$h^2=$	0·0000094	0·3242313	0·1341757	0·1117431	0·0244735
Central Part	....	0·0000314 g	0·0000130 g	0·0000120 g	0·0000024 g
Zone 1	....	0·0000159 „	0·0000066 „	0·0000055 „	0·0000012 „
„ 2	....	0·0000053 „	0·0000022 „	0·0000018 „	0·0000004 „
„ 3	....	0·0000026 „	0·0000011 „	0·0000009 „	0·0000002 „
„ 4	....	....	0·0000007 „	0·0000005 „	0·0000001 „
„ 5	....	....	0·0000005 „	0·0000003 „	0·0000001 „
„ 6	....	....	....	0·0000002 „	0·0000001 „
„ 7	....	....	....	0·0000001 „	....
„ 8	....	....	....	0·0000001 „	....
„ 9	....	....	....	0·0000001 „	....
„ 10	....	....	....	....	....
„ 11	....	....	....	....	....
„ 12	....	....	....	....	....
„ 13	....	....	....	....	....
„ 14	....	....	....	....	....
Totals, $m=$ 50...	....	0·0000552 g	0·0000241 g	0·0000215 g	0·0000045 g

For $m=100$ it will be quite near enough to double these; viz.

Totals, $m=100$...	....	0·0001104 g	0·0000482 g	0·0000430 g	0·0000090 g

16. Suppose on a zone of any width only a comparatively small portion of its whole circuit has a mass standing on it. Then, as the distance from the centre of the mass increases, the angular width varies nearly as the distance inversely. By paragraph 11 the Vertical Attraction of the mass on the whole zone, t being its thickness,

$$=\frac{3g}{4c}\frac{w-u}{2c}\left(t+\frac{t^2c}{uw}\right).$$

From which it is easy to deduce that, for a whole zone,

$$\text{Resultant Vertical Attraction}=-\frac{3g}{4}\frac{w-u}{2c}\frac{(m+1)t^2}{uw}.$$

This varies very nearly inversely as the square of the distance from the centre of the mass. Hence, if the mass stands on only a comparatively small portion of the zone measured horizontally at the station, the Resultant Vertical Attraction varies nearly inversely as the cube of the distance.

17. I will now consider the case of a zone the height of the mass upon it being such that we must not neglect any power of the depth through which the corresponding attenuation reaches.

We must revert to formula (1). The effect of the attenuation below the zone equals the difference of the effects of two masses, each 1-mth of the density of rock, running down to depths h and $h+mk$ below the station-level. Call u and w, as before, the bounding chords of the zone. The effect of the attenuation

$$=-\frac{3g}{4cm}\Big(w-u-\sqrt{w^2+(h+mk)^2}+\sqrt{u^2+(h+mk)^2}$$

$$+\frac{w-u}{2c}(h+mk)-w+u+\sqrt{w^2+h^2}-\sqrt{u^2+h^2}-\frac{w-u}{2c}h\Big).$$

Suppose that the zone is the rth, then

$$w=\frac{2c(r+1)}{n},\quad u=\frac{2cr}{n}.$$

Substituting these, neglecting the fourth power of $h\div u$, and introducing a subsidiary angle ϕ, such that

$$\frac{n(h+mk)}{c(2r+1)}=\tan\phi,$$

the effect of the attenuation

$$=\frac{3g}{4nm}\left\{-2+\sqrt{4(r+1)^2+(2r+1)^2\tan^2\phi}-\sqrt{4r^2+(2r+1)^2\tan^2\phi}-\frac{mk}{c}+\frac{n^2h^2}{4r(r+1)c^2}\right\}.$$

The pair of radicals in this expression

$$=\sqrt{1+(2r+1)^2\sec^2\phi+2(2r+1)}-\sqrt{1+(2r+1)^2\sec^2\phi-2(2r+1)}$$

$$=\frac{2(2r+1)}{(1+(2r+1)^2\sec^2\phi)^{\frac{1}{2}}}+\frac{(2r+1)^3}{(1+(2r+1)^2\sec^2\phi)^{\frac{5}{2}}}+\ldots \text{ by expansion}$$

$$=2\cos\phi\left(1-\frac{\cos^2\phi}{2(2r+1)^2}\right)+\frac{\cos^5\phi}{(2r+1)^2}+\ldots$$

$$=2\cos\phi-\frac{\cos^3\phi-\cos^5\phi}{(2r+1)^2}\text{ nearly }=2\cos\phi-\frac{2\cos\phi-\cos 3\phi-\cos 5\phi}{16(2r+1)^2}.$$

substituting this in the expression for the effect of the attenuation, and adding it to the vertical attraction of the mass above the sea-level given in formula (3),

$$\text{Resultant Vertical Attraction for the zone}=\frac{3g}{2mn}\,\frac{\beta}{360}\,\mathrm{R},$$

where β is the angular extent, at the station, of the part of the zone on which attracting matter stands, at the height k; and R is given by the following formula:—

$$\mathrm{R}=-1+\cos\phi-\frac{2\cos\phi-\cos 3\phi-\cos 5\phi}{32(2r+1)^2}-\frac{n^2}{8c^2}\,\frac{m(k^2-2hk)-h^2}{r(r+1)},\text{ and }\tan\phi=\frac{h+mk}{2r+1}\,\frac{n}{c}.$$

The expression for R may be somewhat simplified for zones beyond a certain distance. For when ϕ is sufficiently small to allow of its fourth power being neglected,

$$\phi^2=\frac{n^2}{c^2}\left(\frac{h+mk}{2r+1}\right)^2,$$

and the part of R depending on ϕ becomes by expansion

$$-\frac{\phi^2}{2}\left(1+\frac{1}{(2r+1)^2}\right)+\frac{\phi^4}{24}\left(1+\frac{22}{(2r+1)^2}\right).$$

Neglecting ϕ^4, and substituting for ϕ^2, this becomes

$$-\frac{n^2}{2c^2}\left(\frac{h+mk}{2r+1}\right)^2\left(1+\frac{1}{(2r+1)^2}\right),$$

and

$$\mathrm{R}=-\frac{n^2}{2c^2}\left\{\frac{(h+mk)^2}{(2r+1)^2}\left(1+\frac{1}{(2r+1)^2}\right)+\frac{m(k^2-2kh)-h^2}{4r(r+1)}\right\}.$$

In order to ascertain for what zones this simpler formula for R may be used, I observe that in the final result decimals are to be retained to the 7th place in the ratio of vertical attraction to gravity. Hence $3\beta R \div 720nm$ must be calculated to seven places of decimals. The largest value β will have is 149°. Hence in $447R \div 720nm$ a quantity as small as 0·0000001 must be retained, or a quantity in R as small as $0{\cdot}000000161nm$, or say $0{\cdot}00000016nm$, must be retained. Hence the neglected term

$$\frac{\phi^4}{24}\left(1+\frac{22}{(2r+1)^2}\right) \text{ or } \frac{n^4}{24c^4}\left(\frac{h+mk}{2r+1}\right)^4\left(1+\frac{22}{(2r+1)^2}\right) \text{ must be } < 0{\cdot}00000016nm,$$

or

$$2r+1 > \frac{n(h+mk)}{174}\sqrt[4]{\frac{1}{nm}\left(1+\frac{22}{(2r+1)^2}\right)}.$$

When numerical values are given to the quantities involved it will be easy to find the least integral value of r which satisfies this condition; that value of r shows the first zone for which the second form of R can be used.

18. I purpose making, as I have already said, $n=160$. The several formulæ now calculated in the last paragraph I gather together and write down here, n being put$=160$.

$$\text{Resultant Vertical Attraction for zone} = \frac{g\beta}{38400}\frac{R}{m}. \quad \ldots \quad (4)$$

$$R = -1+\cos\phi - \frac{2\cos\phi - \cos 3\phi - \cos 5\phi}{32(2r+1)^2} - 0{\cdot}00020\,\frac{m(k^2-2kh)-h^2}{r(r+1)}, \quad \ldots \quad (5)$$

$$\tan\phi = 0{\cdot}04045\,\frac{h+mk}{2r+1},$$

or

$$R = -0{\cdot}00082\left(\frac{(h+mk)^2}{(2r+1)^2}+\frac{(h+mk)^2}{(2r+1)^4}+\frac{m(k^2-2kh)-h^2}{4r(r+1)}\right), \quad \ldots \quad (6)$$

when

$$2r+1 \text{ is} > \frac{h+mk}{3{\cdot}8}\left[\frac{1}{m}\left(1+\frac{22}{(2r+1)^2}\right)\right]^{\frac{1}{4}}. \quad \ldots \quad (7)$$

§ 5. *Numerical application of the formulæ to find the "Resultant Vertical Attraction" of the Himalayas upon Stations of the Indian Arc of Meridian through Cape Comorin.*

19. The formulæ of the last paragraph I shall now apply to find the resultant vertical attraction of the Himalayas at the three nearest of the stations I have entered in the Table in § 1, viz. Kaliana, Kalianpur, Damargida. The attraction at the rest can be found more simply. I shall take two cases of m, viz. $m=50$ and $m=100$.

FIRST, $m=50$.

Station Kaliana.

$$h=0\cdot1564,\ k=2\cdot8409,\ h+mk=142\cdot202.$$
$$h^2=0\cdot02446,\ k^2-2kh=7\cdot18208,\ (h+mk)^2=20220\cdot84.$$

By (7) we have $2r+1=15$, $r=7$. Hence formula (5) must be used up to the 6th zone; after that formula (6) up to the 17th or last.

$$\tan\phi=0\cdot04045\frac{h+mk}{2r+1}=\frac{5\cdot75207}{2r+1}.$$

r.	$\tan\phi$.	ϕ.	$\cos\phi$.	$2\cos\phi-\cos3\phi-\cos5\phi=$	P.
2	1·15041	49° 0′	0·65606	1·31212+0·83867+0·42262	2·57341
3	0·82172	39 25	0·77255	1·54510+0·47332+0·95757	2·57599
4	0·63912	32 35	0·84261	1·68522+0·13485+0·95588	2·77595
5	0·52273	27 36	0·88620	1·77240−0·12533+0·74314	2·39021
6	0·44247	23 52	0·91449	1·82898−0·31565+0·48989	2·00322

By (4) and (5) $R=-1+\cos\phi-\frac{P}{32(2r+1)^2}-\frac{0\cdot07182}{r(r+1)}$, $\text{Resultant}=\frac{\beta}{1920000}Rg$.

r.	Values of R.	β.	Resultants.
2	−0·34394−0·00322−0·01197=−0·35913	74°	−0·0000138 g
3	−0·22745−0·00189−0·00598=−0·23532	110	−0·0000135 „
4	−0·15739−0·00108−0·00359=−0·16206	127	−0·0000107 „
5	−0·11380−0·00063−0·00239=−0·11682	137	−0·0000083 „
6	−0·08551−0·00037−0·00171=−0·08759	144	−0·0000066 „

By (6) $R=\frac{-16\cdot58107}{(2r+1)^2}-\frac{16\cdot58107}{(2r+1)^4}-\frac{0\cdot07362}{r(r+1)}$.

7	−0·07369−0·00033−0·00131=−0·07533	149	−0·0000058 g
8	−0·05737−0·00020−0·00102=−0·05859	113	−0·0000034 „
9	−0·04593−0·00013−0·00082=−0·04688	88	−0·0000021 „
10	−0·03760−0·00009−0·00067=−0·03836	79	−0·0000016 „
11	−0·03134−0·00006−0·00056=−0·03196	66	−0·0000011 „
12	−0·02653−0·00004−0·00047=−0·02704	56	−0·0000008 „
13	−0·02274−0·00003−0·00040=−0·02317	109	−0·0000013 „
14	−0·01972−0·00002−0·00035=−0·02009	87	−0·0000009 „
15	−0·01725−0·00002−0·00031=−0·01758	79	−0·0000007 „
16	−0·01523−0·00002−0·00027=−0·01552	61	−0·0000005 „
17	−0·01354−0·00001−0·00024=−0·01379	37	−0·0000003 „
Resultant at Kaliana for the Himalayas			−0·0000714 g

Station Kalianpur.

$$h=0\cdot3363,\ k=2\cdot84091,\ h+mk=142\cdot3818.$$
$$h^2=0\cdot11310,\ k^2-2kh=6\cdot15992,\ (h+mk)^2=20272\cdot58.$$

By (7) we have $2r+1=15$, and $r=7$. Hence only formula (6) need be used, because the first zone in this case is the 8th.

$$R=-\frac{16\cdot62352}{(2r+1)^2}-\frac{16\cdot62352}{(2r+1)^4}-\frac{0\cdot06312}{r(r+1)}.$$

r.	Values of R.	β.	Resultants.
8	−0·05752−0·00020−0·00088=−0·05860	45°	−0·0000014 g
9	−0·04605−0·00013−0·00063=−0·04681	62	−0·0000015 "
10	−0·03769−0·00009−0·00057=−0·03835	79	−0·0000016 "
11	−0·03141−0·00006−0·00048=−0·03195	88	−0·0000015 "
12	−0·02660−0·00004−0·00040=−0·02704	92	−0·0000013 "
13	−0·02280−0·00003−0·00035=−0·02318	97	−0·0000012 "
14	−0·01976−0·00002−0·00030=−0·02008	76	−0·0000008 "
15	−0·01730−0·00002−0·00026=−0·01758	58	−0·0000005 "
16	−0·01526−0·00001−0·00023=−0·01550	47	−0·0000004 "
17	−0·01357−0·00001−0·00021=−0·01379	46	−0·0000003 "
18	−0·01214−0·00001−0·00018=−0·01233	35	−0·0000002 "
19	−0·01093−0·00001−0·00017=−0·01111	76	−0·0000004 "
20	−0·00989−0·00001−0·00015=−0·01005	46	−0·0000002 "
21	−0·00899 ....−0·00014=−0·00913	6	....
22	−0·00821 ....−0·00012=−0·00833	6	....
	Resultant at Kalianpur for the Himalayas		−0·0000113 g

Station Damargida.

$$h=0\cdot3663,\ k=2\cdot84091,\ h+mk=142\cdot4118.$$
$$h^2=0\cdot13418,\ k^2-2kh=5\cdot98947,\ (h+mk)^2=20281\cdot13.$$

By (7) we have $2r+1=15$, $r=7$. Hence we need use only the formula (6).

$$R=-\frac{16\cdot63052}{(2r+1)^2}-\frac{0\cdot06136}{r(r+1)}.$$

r.	Values of R.	β.	Resultants.
17	− 0·01358 − 0·00020 = − 0·01378	22°	−0·0000002 g
18	− 0·01215 − 0·00018 = − 0·01233	41	−0·0000003 "
19	− 0·01093 − 0·00016 = − 0·01109	50	−0·0000003 "
20	− 0·00989 − 0·00015 = − 0·01004	62	−0·0000003 "
21	− 0·00899 − 0·00013 = − 0·00912	68	−0·0000003 "
22	− 0·00865 − 0·00012 = − 0·00877	73	−0·0000003 "
23	− 0·00753 − 0·00011 = − 0·00764	58	−0·0000002 "
24	− 0·00693 − 0·00010 = − 0·00703	46	−0·0000002 "
25	− 0·00639 − 0·00009 = − 0·00648	32	−0·0000001 "
26	− 0·00592 − 0·00008 = − 0·00600	9	....
27	− 0·00550 − 0·00008 = − 0·00558	12	....
28	− 0·00512 − 0·00008 = − 0·00520	12	....
29	− 0·00477 − 0·00007 = − 0·00484	12	....
	Resultant at Damargida for the Himalayas.........		−0·0000022 g

SECONDLY, $m=100$.

Station Kaliana.

$h=0{\cdot}1564,\ k=2{\cdot}84091,\ h+mk=284{\cdot}247.$

$k^2=0{\cdot}02446,\ k^2-2kh=7{\cdot}18208,\ (h+mk)^2=80790{\cdot}68.$

By (7) we have $2r+1=25$, $r=12$. Hence up to the 11th zone we must use formula (5) and after that (6).

$$\tan\phi=0{\cdot}04045\,\frac{h+mk}{2r+1}=\frac{11{\cdot}49779}{2r+1}.$$

r.	$\tan\phi$.	ϕ.	$\cos\phi$.	$2\cos\phi-\cos 3\phi-\cos 5\phi=$	P.
2	2·29956	66° 30′	0·39875	0·79750+0·94264−0·88701	0·85313
3	1·64254	58 40	0·52002	1·04004+0·99756−0·39608	1·64152
4	1·27753	51 53	0·61726	1·23452+0·91104+0·18367	2·32923
5	1·04525	46 16	0·69130	1·38260+0·75241+0·62479	2·75980
6	0·88444	41 29	0·74915	1·49830+0·56569+0·88768	2·95167
7	0·76652	37 28	0·79371	1·58742+0·33107+0·99182	2·91031
8	0·67634	34 4	0·82839	1·65678+0·21132+0·98580	2·85390
9	0·60515	31 11	0·85551	1·71102+0·06192+0·91295	2·68589
10	0·54751	28 42	0·87715	1·75430−0·06802+0·80386	2·49014
11	0·49990	26 34	0·89441	1·78882−0·17880+0·67987	2·28989

By (4) and (5) $\mathrm{R}=-1+\cos\phi-\dfrac{\mathrm{P}}{32(2r+1)^2}-\dfrac{0{\cdot}14364}{r(r+1)}$, and Resultant $=\dfrac{\beta}{3840000}\,\mathrm{R}$.

r.	Values of R.	β.	Resultants.
2	−0·60125−0·00106−0·02394=−0·62625	74°	−0·0000120 g
3	−0·47998−0·00105−0·01193=−0·49296	110	−0·0000141 ,,
4	−0·38274−0·00095−0·00728=−0·39097	127	−0·0000130 ,,
5	−0·30870−0·00072−0·00479=−0·31421	137	−0·0000112 ,,
6	−0·25085−0·00054−0·00342=−0·25481	144	−0·0000096 ,,
7	−0·20629−0·00045−0·00256=−0·20930	149	−0·0000081 ,,
8	−0·17161−0·00031−0·00199=−0·17391	113	−0·0000051 ,,
9	−0·14449−0·00025−0·00160=−0·14634	88	−0·0000034 ,,
10	−0·12285−0·00019−0·00131=−0·12435	79	−0·0000026 ,,
11	−0·10559−0·00014−0·00109=−0·10682	66	−0·0000018 ,,

$$\text{By (6) }\mathrm{R}=-\frac{66{\cdot}24836}{(2r+1)^2}-\frac{66{\cdot}24836}{(2r+1)^4}-\frac{0{\cdot}14723}{r(r+1)}.$$

12	−0·10600−0·00017−0·00094=−0·10711	56	−0·0000016 g
13	−0·09087−0·00013−0·00081=−0·09181	109	−0·0000030 ,,
14	−0·07877−0·00010−0·00070=−0·07957	87	−0·0000020 ,,
15	−0·06894−0·00010−0·00061=−0·06965	79	−0·0000014 ,,
16	−0·06083−0·00005−0·00054=−0·06142	61	−0·0000010 ,,
17	−0·05509−0·00004−0·00048=−0·05561	37	−0·0000005 ,,
	Resultant at Kaliana for the Himalayas		−0·0000904 g

Station Kalianpur.

$$h=0\cdot3363,\ k=2\cdot84091,\ h+mk=284\cdot427.$$

$$h^2=0\cdot11310,\ k^2-2kh=6\cdot15992,\ (h+mk)^2=80809\cdot42.$$

By (7) we have, as before, $r=12$; and we must use formula (5) to the 11th zone, and after that formula (6).

$$\tan\phi=0\cdot04045\,\frac{h+mk}{2r+1}=\frac{11\cdot50587}{2r+1}.$$

r.	tan ϕ.	ϕ.	cos ϕ.	2 cos ϕ − cos 3ϕ − cos 5ϕ =	P.
8	0·67681	34° 6′	0·82806	1·65612+0·21303+0·98629=	2·85544
9	0·60556	31 12	0·85536	1·71072+0·06279+0·91355=	2·68706
10	0·54790	28 43	0·87701	1·75402−0·06714+0·80472=	2·49160
11	0·50026	26 35	0·89428	1·79856−0·17794+0·68093=	2·30155

$$\text{By (5) } \mathrm{R}=-1+\cos\phi-\frac{\mathrm{P}}{32(2r+1)^2}-\frac{0\cdot12318}{r(r+1)}.$$

r.	Values of R.	β.	Resultants.
8	−0·17194−0·00031−0·00171=−0·17396	45°	−0·0000020 g
9	−0·14464−0·00024−0·00137=−0·14625	62	−0·0000024 ,,
10	−0·12299−0·00017−0·00112=−0·12428	79	−0·0000026 ,,
11	−0·10572−0·00014−0·00093=−0·10679	88	−0·0000024 ,,

$$\text{By (6) } \mathrm{R}=-\frac{66\cdot26372}{(2r+1)^2}-\frac{66\cdot26372}{(2r+1)^4}-\frac{0\cdot12626}{r(r+1)}.$$

12	−0·10602−0·00017−0·00081=−0·10700	92	−0·0000026 g
13	−0·09090−0·00013−0·00069=−0·09172	97	−0·0000023 ,,
14	−0·07879−0·00009−0·00060=−0·07948	76	−0·0000016 ,,
15	−0·06895−0·00007−0·00053=−0·06955	53	−0·0000010 ,,
16	−0·06085−0·00005−0·00046=−0·06136	47	−0·0000008 ,,
17	−0·05409−0·00004−0·00041=−0·05454	46	−0·0000007 ,,
18	−0·04840−0·00004−0·00037=−0·04881	35	−0·0000005 ,,
19	−0·04357−0·00003−0·00033=−0·04393	76	−0·0000009 ,,
20	−0·03941−0·00002−0·00030=−0·03973	46	−0·0000005 ,,
21	−0·03584−0·00002−0·00027=−0·03613	6	−0·0000001 ,,
22	−0·03272−0·00002−0·00023=−0·03297	6	−0·0000001 ,,
	Resultant at Kalianpur for the Himalayas	....	−0·0000205 g.

Station Damargida.

$$h=0\cdot3663,\ k=2\cdot84091,\ h+mk=284\cdot457.$$
$$h^2=0\cdot13418,\ k^2-2kh=5\cdot98947,\ (h+mk)^2=80915\cdot78.$$

By (7) we have, as before, $r=12$; and therefore formula (6) may be used throughout.

$$\text{By (6) } \mathrm{R}=-\frac{66\cdot35093}{(2r+1)^2}-\frac{0\cdot12278}{r(r+1)}.$$

r.	Values of R.	β.	Resultants.
17	$-$ 0·05416 $-$ 0·00040 = $-$ 0·05456	22°	$-0\cdot0000003\,g$
18	$-$ 0·04847 $-$ 0·00036 = $-$ 0·04883	41	$-0\cdot0000005$,,
19	$-$ 0·04208 $-$ 0·00032 = $-$ 0·04240	50	$-0\cdot0000006$,,
20	$-$ 0·03947 $-$ 0·00029 = $-$ 0·03976	62	$-0\cdot0000006$,,
21	$-$ 0·03588 $-$ 0·00026 = $-$ 0·03614	68	$-0\cdot0000006$,,
22	$-$ 0·03276 $-$ 0·00024 = $-$ 0·03300	73	$-0\cdot0000006$,,
23	$-$ 0·03004 $-$ 0·00022 = $-$ 0·03022	58	$-0\cdot0000005$,,
24	$-$ 0·02763 $-$ 0·00020 = $-$ 0·02783	46	$-0\cdot0000003$,,
25	$-$ 0·02669 $-$ 0·00019 = $-$ 0·02688	32	$-0\cdot0000002$,,
26	$-$ 0·02362 $-$ 0·00018 = $-$ 0·02380	9	$-0\cdot0000001$,,
27	$-$ 0·02184 $-$ 0·00017 = $-$ 0·02201	12	$-0\cdot0000001$,,
28	$-$ 0·02042 $-$ 0·00016 = $-$ 0·02058	12	$-0\cdot0000001$,,
29	$-$ 0·01906 $-$ 0·00015 = $-$ 0·01921	12	$-0\cdot0000001$,,
	Resultant at Damargida for the Himalayas		$-0\cdot0000046\,g$

20. As the Resultant Vertical Attraction at Damargida, which is far from the Himalayas, is small, we may find that at the still further stations in the list, viz. Bangalore and Punnae and the others, by the law of the inverse cube, which I have proved in paragraph 17. The distance of Damargida from the centre of the tableland which I have taken to represent the Himalayas in this problem, is about 16°. Hence the Resultant Vertical Attraction caused by the Himalayas will be as follows:—

	$m=50$.	$m=100$.		$m=50$.	$m=100$.
At Bangalore..............	$-0\cdot0000010\,g$	20 g	At Mangalore	$-0\cdot0000009\,g$	18 g
,, Punnae.................	$-0\cdot0000005$,,	11 ,,	,, Alleppy.................	$-0\cdot0000006$,,	13 ,,
,, Cocanada..............	$-0\cdot0000019$,,	38 ,,	,, Minicoy	$-0\cdot0000005$,,	11 ,,
,, Madras..................	$-0\cdot0000010$,,	20 ,,			

§ 6. *Calculation of the "Resultant Vertical Attraction" of the Sea at Stations on a Continent, on a Coast, or on an Island.*

21. I will suppose a straight coast-line, the sea-bottom shelving down and then rising again, so as practically to be equivalent to a uniform descent to a depth H at a distance U from the shore, and beyond that to have no sensible effect on the Resultant Vertical Attraction.

Let u be the horizontal distance from the shore, and z the depth of an elementary horizontal prism of sea-water, of indefinite length, running parallel to the coast-line; a the distance on the sea-level of a station in the interior of the continent on which the effect of the sea is to be found. Suppose the horizontal prism to consist of two parts of indefinite length, divided at the point opposite the station on the coast. The attraction

of each of these towards the point of division equals the mass of each divided by the product of the distances of its extremities from the station. Hence the deficiency of vertical attraction of the sea on the station in question

$$=-\frac{3g}{4c}\,0\cdot637\int_0^{\mathrm{U}}\int_0^z\frac{zdudz}{z^2+(u+a)^2}=\frac{-1\cdot9g}{8c}\int_0^{\mathrm{U}}\log_e\left(\frac{z^2+(u+a)^2}{(u+a)^2}\right)du.$$

Suppose matter equal in amount to the deficiency in the ocean is spread down uniformly through a depth $m\,.\,z$. Then

$$\text{Resultant Vertical Attraction}=\frac{1\cdot9g}{8c}\int_0^{\mathrm{U}}\left(\frac{1}{m}\log_e\frac{m^2z^2+(u+a)^2}{(u+a)^2}-\left(1+\frac{1}{m}\right)\log_e\frac{z^2+(u+a)^2}{(u+a)^2}\right)du$$

$$=\frac{1\cdot9g}{8c}\int_0^{\mathrm{U}}\frac{(m-1)z^2du}{(u+a)^2}\text{ nearly, }=\frac{1\cdot9g}{8c}\frac{\mathrm{H}^2}{\mathrm{U}^2}(m-1)\int_0^{\mathrm{U}}\frac{u^2}{(u+a)^2}du$$

$$=\frac{1\cdot9g}{8c}\frac{\mathrm{H}^2}{\mathrm{U}^2}(m-1)\mathrm{U}\left\{1-\frac{2a}{\mathrm{U}}\log_e\frac{\mathrm{U}+a}{a}+\frac{a}{\mathrm{U}+a}\right\}.$$

This formula I will apply to find the effect of the sea upon our stations. I will suppose that the bottom shelves down at the same angle on an average on the east and the west coasts, so as to make $\mathrm{H}\div\mathrm{U}=1\div600$, but that on the east coast $\mathrm{U}=600$, and on the west 900 miles.

The distances of the five stations from the east coast are about 0, 180, 330, 560, 840 miles; and from the west coast about 0, 190, 300, 480, 700 miles. The distances of Alleppy and Mangalore from the east coast are about 120 and 350 miles; and of Madras and Cocanada from the west coast about 350 and 560. For these the formula gives as follows:—

TABLE VI.

Stations.	East Coast, $m=50$.	East Coast, $m=100$.	West Coast, $m=50$.	West Coast, $m=100$.	Totals, $m=50$.	Totals, $m=100$.
Punnae	0·0000050 g	0·0000100 g	0·0000077 g	0·0000154 g	0·0000127 g	0·0000254 g
Bangalore	0·0000018 ,,	0·0000036 ,,	0·0000032 ,,	0·0000064 ,,	0·0000050 ,,	0·0000100 ,,
Damargida	0·0000010 ,,	0·0000020 ,,	0·0000024 ,,	0·0000048 ,,	0·0000034 ,,	0·0000068 ,,
Kalianpur	0·0000006 ,,	0·0000012 ,,	0·0000016 ,,	0·0000032 ,,	0·0000022 ,,	0·0000044 ,,
Kaliana	0·0000003 ,,	0·0000006 ,,	0·0000012 ,,	0·0000024 ,,	0·0000015 ,,	0·0000030 ,,
Punnae	0·0000050 ,,	0·0000100 ,,	0·0000100 ,,	0·0000200 ,,	0·0000127 ,,	0·0000254 ,,
Alleppy	0·0000022 ,,	0·0000044 ,,	0·0000044 ,,	0·0000088 ,,	0·0000099 ,,	0·0000198 ,,
Mangalore	0·0000010 ,,	0·0000020 ,,	0·0000020 ,,	0·0000040 ,,	0·0000087 ,,	0·0000174 ,,
Madras	0·0000050 ,,	0·0000100 ,,	0·0000100 ,,	0·0000200 ,,	0·0000072 ,,	0·0000144 ,,
Cocanada	0·0000050 ,,	0·0000100 ,,	0·0000100 ,,	0·0000200 ,,	0·0000066 ,,	0·0000132 ,,

22. I will now take the case of an Island, and suppose it to be in the form of a cylinder of radius a in a sea of uniform depth h. Then by formula (1) we easily obtain the following result:—

Resultant Vertical Attraction at the middle of the Island

$$=\frac{1\cdot9g}{4c}\left\{\frac{u+(1+m)h-\sqrt{u^2+(1+m)^2h^2}}{m}-\frac{1+m}{m}(u+h-\sqrt{u^2+h^2})\right.$$

$$\left.-\frac{a+(1+m)h-\sqrt{a^2+(1+m)^2h^2}}{m}+\frac{1+m}{m}(a+h-\sqrt{a^2+h^2})\right\}$$

$$=\frac{1\cdot9g}{4c}\left\{(a+h-\sqrt{a^2+h^2})\frac{1+m}{m}-\left(a-\frac{a^2}{2(m+1)h}\right)\frac{1}{m}\right\},\ u\text{ being large.}$$

Minicoy Island is about 250 miles west of Punnae. Its average radius is about 2·5 miles; and three miles from the shore the sea is about 300 fathoms, or about one-third of a mile. If, then, to make our cylinder accord with this case, right angles being cut down, we make $a=5$ and $h=1$, we have

	$m=50$.	$m=100$.
Minicoy Island.........	0·0000988 g	0·0001046 g

§ 7. *Application of these results to test the truth of the author's hypothesis regarding the Constitution of the Earth's Crust.*

23. In the following Table I bring together the several data and results of this paper in order to compare them.

TABLE VII.

Stations.	Relative gravity freed from effects of height and latitude.	Correction for Local Attraction.							Differences of Gravity.			
		By Dr. Young's formula.	By the present hypothesis.						Actual effects of local attraction.	Corrections of local attraction.		
			$m=50$.			$m=100$.				By Dr. Young.	By this hypothesis.	
			Plains.	Himalayas.	Sea.	Plains.	Himalayas.	Sea.			$m=50$.	$m=100$.
Indian Arc Stations.										A.	B.	C.
Punnae	1−0·0006094	− 14	......	+ 5	−127	......	+ 11	− 254	....	....	....	....
Bangalore	1−0·0005710	−960	−552	+ 10	− 50	−1104	+ 20	− 100	+384	−946	−471	− 941
Damargida	1−0·0006417	−617	−241	+ 22	− 34	− 482	+ 46	− 68	−323	−603	−132	− 261
Kalianpur	1−0·0005753	−563	−215	+113	− 22	− 430	+205	− 44	+341	−549	− 3	− 26
Kaliana	1−0·0006801	−264	− 45	+714	− 15	− 90	+904	− 30	−707	−250	+776	+1027
Coast Stations.												
Punnae	1−0·0006094	− 14	......	+ 5	−127	......	+ 11	− 254	....	....	....	....
Alleppy	1−0·0005792	− 2	......	+ 6	− 99	......	+ 13	− 198	+302	+ 12	+ 29	+ 58
Mangalore.........	1−0·0006260	− 2	......	+ 9	− 87	......	+ 18	− 174	−166	+ 12	+ 44	+ 87
Madras	1−0·0006291	− 9	......	+ 10	− 72	......	+ 20	− 144	−197	+ 5	+ 59	+ 119
Cocanada	1−0·0005952	− 3	......	+ 19	− 66	......	+ 38	− 132	+142	+ 11	+ 74	+ 149
Ocean Station.												
Minicoy............	1−0·0005200	− 2	......	+ 5	−988	......	+ 11	−1046	+894	+ 12	−863	− 792

Under the heading "Correction for Local Attraction" Dr. YOUNG'S is found by taking one third for the "Reduction for Height" in Table III. The three columns for Plains, Himalayas, and Sea are taken from paragraphs 15, 19, 21, 22. These three columns, in each case of m, are added together, and after the values for Punnae are subtracted the results are recorded in columns B and C.

In order the better to compare these results with the quantities which are to be accounted for, I compile the following Table from the columns A, B, C of Table VII. The sign + indicates a force acting downwards. In all the columns the numbers are the last figures of seven places of decimals in the ratio to gravity, the decimal point and ciphers being omitted for convenience.

TABLE VIII.

Stations.	Differences of Gravity.			
	Relative effects of local attraction deduced from Pendulum Observations.	Residual errors after correction by the methods of		
		Dr. YOUNG.	This hypothesis.	
			$m=50$.	$m=100$.
Indian Arc Stations.				
Punnae	....	....	....	....
Bangalore	+384	−562	− 78	−557
Damargida	−323	−926	−455	−584
Kalianpur	+341	−208	+338	+315
Kaliana	−707	−957	+ 69	+320
Coast Stations.				
Punnae	....	....	....	....
Alleppy	+302	+314	+331	+360
Mangalore	−166	−154	−122	− 79
Madras	−197	−192	−138	− 78
Cocanada	+142	+153	+216	+291
Ocean Station.				
Minicoy	+894	+ 906	+ 31	+102

24. This Table contains all the final results necessary to enable us to judge of the truth of the hypothesis which I advocate in this paper, and which I will now discuss.

From the first column of numbers we learn that, according to Pendulum Observations, gravity at the four stations I have chosen north of Punnae, when every cause of variation is eliminated except Local Attraction, is alternately in excess and defect of that at Punnae, the first (at Bangalore) being in excess. In the whole range of stations in my list, the effect on gravity at Kaliana and Minicoy is the most important—the first in defect, the last in excess.

I will now consider how far the effects exhibited in this Table are accounted for on Dr. YOUNG's (or the usually received) method, and my own.

The Coast stations shall first be taken. In these neither method has much success in accounting for the local attraction. A survey of the form of the land and sea-bottom near those places would very likely change this result. The local vertical attraction at Alleppy is the greatest, and is in excess. This may be accounted for probably in part by the sea between it and Minicoy being *deeper* than the general slope which I have assumed in the calculation, as will be understood when I come to refer to Minicoy. None of the local vertical attractions at the coast stations are *very* large.

Next let us take the Indian Arc and Ocean Stations. The second column of numbers shows that Dr. YOUNG's correction, so far from improving matters, introduces very large residual errors, and those on the Arc are all in the same direction. And I may add that, if his method of allowing for the effect of all superficial causes of disturbance is fully carried out, a negative quantity must be added to correct the effect of the Himalayas at Kaliana, as they are below its horizon, and a positive quantity at Punnae for the sea. And when all are referred to Punnae the whole series of numbers for this mode of correction, which disregards the state of the interior, is even greater than before. The third and fourth columns show the effect of the method of this paper, in the two cases of the

compensation below running down 50 and 100 times the heights above the sea-level and depths of the sea below that level. The first of these gives the best results. This, then, I shall discuss. The only residual errors which are large are those at Damargida, showing a defect in gravity at that place compared with Punnae, and at Kalianpur in excess. On examining the numbers in the last column of Table I., paragraph 3, it will be seen that there is a very small horizontal force at Damargida, and at the stations next to it, north and south, the horizontal force is directed *from* Damargida in both cases. This indicates a deficiency of matter in the neighbourhood of that station, which accords with the residual error in my last Table VIII. Also in Table I. we see that there is a horizontal force from Kalianpur towards Takal Khera, and also from Takal Khera towards Kalianpur. This indicates an abnormal excess of matter between those stations, and nearer to Kalianpur than to Takal Khera, as the force at the former is the larger of the two. This accords with an excess of matter near to Kalianpur, which is indicated by an excess of gravity shown in my Table VIII. At Bangalore the Table indicates a slight abnormal deficiency of matter. But Table I. shows a north horizontal force at Dodagoontah, more than four miles south of Bangalore. This would seem to imply that the slight deficiency of matter which causes the defect of gravity at Bangalore runs further south of Dodagoontah than it does north. It is generally difficult to compare the horizontal and vertical effects of a hypothetical excess or defect of matter, as all depends upon its situation relatively to the stations. Thus an excess or defect immediately below a station will not affect the plumb-line, whereas a defect or excess near the surface, and between stations and far from both, will not affect the vertical force materially. On the whole the peculiarities at Damargida, Kalianpur, and Bangalore seem to be sufficiently accounted for. The other residual errors in Table VIII., at Kaliana and Minicoy, are so small that they may be considered evanescent; and seeing that the local attractions at those places, as shown in the first column of Table VIII., are very large, this result speaks decidedly in favour of the hypothesis. The fact that the anomalous circumstance is accounted for by the hypothesis, that a station out at sea exhibits a considerable increase in gravity, although surrounded by the ocean, which has a deficiency of attracting matter, is a very strong argument in favour of the hypothesis. Were the exact contour of the continent and the neighbouring sea-bed better known, the application of this method might be carried out more completely. As it is, however, what remains unexplained is not important.

When we remember, then, that the calculations have been conducted in the particular case of the distribution of matter, in excess and defect, being *uniform*, whereas in the contracting of the mass this is not at all likely to have strictly been the case, and observe the way in which the hypothesis nearly explains the errors, while the usually received method does not do so at all, but indeed aggravates them considerably, I think the hypothesis may be regarded as receiving support from the Pendulum Observations recently made on the extensive continent, coast, and (in one important instance) the neighbouring sea of India.